Case: Choosing and Ordering Student Work

Overview

Among the many skills learned in the first few semesters of teaching are (a) observing students as they work on mathematics problems and (b) deciding how to (re)shape teaching based on the information gleaned from such observations. This case offers a glimpse into the geometry teaching apprenticeship of a novice college mathematics instructor. Case activities focus on the development of instructor knowledge of, and response to, student thinking. In particular, the case shows the techniques used by the mentor instructor and TA mentee as each observes students working in groups and then orchestrates presentations of results by those groups.

Learning Goals for Participants

- Develop awareness of instructor role in observing students as they work in groups.
- Explore ways to foster explanation and justification in classroom conversation about mathematical ideas.

Video Content and Context

The video is from the beginning of a *Geometry for Secondary Teachers* course for each of two instructors: Professor Miller and TA Wakefield. First we see clips from Miller's 1st and 3rd days in the Fall term. During that term, Wakefield was an in-class TA, assisting in Miller's class, learning how to teach using *inquiry-based methods*. The second set of video is from the subsequent Spring semester, Wakefield's 1st and 3rd days, as he is teaching the class on his own for the first time. Both instructors used the viewing tube activity that starts the case materials. While the timeline for each class is compressed, all classroom video is presented in the order in which it occurred. A final vignette is the two instructors discussing their classes.

Activity Timeline ($\approx$ 50 minutes)

Preview: 8 minutes

View 1: 8 minutes

Discuss 1: 6 minutes

View 2: 8 minutes

Discuss 2: 10 minutes

Reflect: 10 minutes

Extension: an additional 14 minutes for EACH potential extension

Leading the Case

New mathematics instructors, including TAs, may have scant experience in guiding or observing students as they work in groups on a mathematical task. Most new TAs have spent a great deal of time solving problems themselves but little time watching others attempt to do so. In the video for this case, Professor Miller is an experienced college instructor with many years of practice in inquiry-based methods (e.g., methods that rely on students working through structured problem situations on their own and then presenting and justifying solutions to the class). The interview that introduces the classroom vignettes and that wraps up the end of the case was conducted at the end of year, after both men had taught the first semester of the two-semester *Geometry for Secondary Teachers* content course.

Introduction and Preview (8 minutes) Let participants know that they have about 8 minutes total to read the introduction and to jot down their responses to the preview question. ↦

One way to support participants in learning from the case is to review the differences among *description*, *explanation*, and *justification*. Overall, the goal is to encourage instructors to own their instructional choices, including the ways one fosters:

description "this is **what** we did, the steps"

explanation "this is **how** we decided to do what we did, based on the problem situation"

justification "this is **why** what we did was appropriate and why we rejected some alternative approach(es)"

Choosing and ordering student work requires one to notice what students communicate about their thinking and pay attention to how that may differ from instructor anticipations. That is, an instructor may intend (or assume) that students will engage with an activity at the level of justification but may not support such engagement (because they are asking questions that elicit description or explanation).

One take-away message for this video case is to be deliberate in orchestrating classroom presentations by students. In particular, one useful strategy is sequencing so that describers and explainers go first and the complexity of conversation is scaffolded up to the justifiers.

After the Preview – Preparing for View 1 Let participants know that the first part of the video is about 6 minutes long and that after talking about the first discussion question, and viewing and discussing the second clip, the group will watch the two video vignettes again. ↦

NOTE: It is important to watch the video at least TWICE so participants have the opportunity to notice multiple layers of information.

Choosing and Ordering Student Work

Learning Goals

- Notice the differences between *description*, *explanation*, and *justification* in how students and instructors communicate about problem-solving.
- Be intentional in decision-making about the order in which student groups present their arguments to scaffold explanation and justification in classroom discussions.

Introduction

Many undergraduates have experience solving problems. However, when called on to "explain" or "justify" the solution, they rely on describing the procedures followed to generate the answer. That is, they have practice with *description* of what they did but little experience with giving an explanation of how they decided to do what they describe and even less experience in generating a well-connected justification for why the chosen solution method was appropriate and/or optimal.

The main chunks of video in this case are from the 1st and 3rd days of *Geometry for Secondary Teachers* for each of two instructors: Professor Miller and TA Wakefield. First we see clips from Miller's 1st and 3rd days in the Fall term, when Wakefield was an in-class TA, assisting in Miller's class, learning how to teach using inquiry-based learning. Then we see, from the subsequent semester, Wakefield's 1st and 3rd days, as he is teaching the class on his own for the first time. Both instructors used the same activity. While the timeline for each class is compressed, all classroom video is presented in the order in which it occurred. The interview that introduces the classroom vignettes and that wraps up the end of the case was conducted at the end of year, after both men had taught the course.

Preview

Suppose you are standing on a level floor and are some distance from a wall (the wall and floor surfaces are perpendicular to each other). You hold up a cylindrical tube so that its length is parallel to the floor and peer through the cylinder at the wall. What is the mathematical relationship between your distance from the wall and the diameter of the viewable disk? See figure.

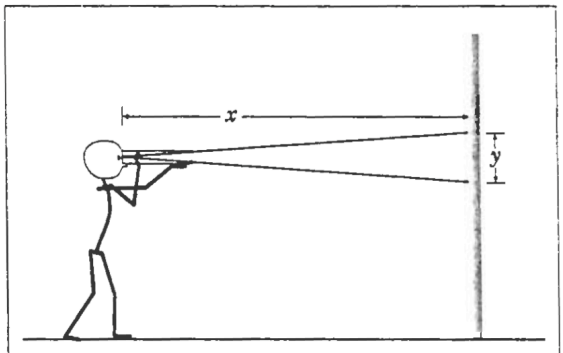

Fig. 1
Original view-tube activity, where x is the distance from the wall and where y is the height of view seen

Source: Keiser, J. M. (2003). Variations on a view tube. *Mathematics Teacher, 96*, 170-176.

View 1 (4 minutes) Before showing video, take a moment to let participants revisit the context information in the introduction and read the first discussion questions. Encourage participants to record their thoughts and observations in the table for each clip. Give a little time immediately after viewing the video to complete writing these notes before starting the discussion.

For the viewing tubes task at the heart of this case, here are some examples of what instructors might say under each heading:

Describing	Explaining	Justifying
What pieces of information did you use?	Show us, explain what you did.	Why did you do this part? How does that connect with what this other group did?
What did you do here?	How did you decide to do that?	So you didn't use the Pythagorean theorem, why not?

Discuss 1 (8 minutes) Focus and push this brief conversation so discussion is about the EVIDENCE from people in the video that supports each statement about what student and professors' perceptions and intentions are and for why they may be similar or different.

Participants may want to keep talk focused on judgements regarding the professors and students (e.g., whether they agree with certain statements). It may be helpful to acknowledge that each person will have different responses and advise participants to note those responses, mentally or on paper, but that the conversational focus is perceptions and intentions of those in the video as evidenced by the video. Keep the discussion to 8 minutes to ensure there is time to re-view the video and to get to the second discussion.

View 2 (9 minutes) Before showing video for the second time, take a moment to let participants (re)read the discussion questions. NOTE: It is important to watch the video at least TWICE so participants have the opportunity to notice multiple layers of information.

Discuss 2 (15 minutes) Again, focus and push conversation so discussion includes warrants and backing for assertions – What is the EVIDENCE from the video that supports each participant assertion.

View 1

Watch the video, paying attention to how the viewing tube activity is being talked about by professors and engaged in by students. Note the wording of questions and of answers. Keep track of which seem to be *description* (e.g., of steps), those that call for or offer *explanation*, and things that are *justification*. The table below is offered as a place for you to make notes about what students and professors say and for your reference in comparing and contrasting their views.

Describing	Explaining	Justifying

Discuss 1

1. Which students seem to be focusing on description? Explanation? Justification? How do you know (what is the evidence, what do they say or do)? What types of things are the instructors saying that are about description? Explanation? Justification?

2. Compare the teacher and student views.

View 2

Professor Miller, when interviewed about his management of the whole-class presentation of solutions, noted that on Day 1, when he had been walking from group to group watching them work and getting familiar with the students, he had paid attention to the complexity of the communication about mathematical ideas he heard in each group and he had created a mental list from least to most complex. Then, on Day 3, he asked groups to report out to the class in the order he had noticed (calling on the least complex within-group discussion first, more complex second, etc., with the last two groups being the ones he had observed as engaging in multiple ways with the activity through reasoning, problem-solving, representing, and connecting). Watch the video again, look for this progression in student presentations.

Discuss 2

How were the experience of the students in the two classes different? Be sure to point to specific evidence in the video/audio of the classes.

Additional Activities

Watch additional video from interviews with instructors and from class and:

1. Explore students' struggles to understand the immediate task and why it is valuable mathematically.
2. Explore instructor aims and compare to class session video for each.
3. Compare and contrast the class session videos in terms of (a) what students have an opportunity to learn and (b) strengths and challenges for student thinking and understanding.

Related Resources

Connections to other cases:

- **Case: Facilitating Group Work** — This case digs more deeply into different approaches to supporting group work done by students.
- **Essay: Questions** — The complexity of questions asked and answered during instruction (and associated wait time) are outlined and illustrated in this essay.

Reflect

1. Watch the last few minutes of the video (from the Discussion screen on to the end). How were your perceptions of each instructor's actions similar to, and different from, those mentioned by the two instructors?

2. Identify some of the pros and cons of the "starting from scratch" idea in the final comment by Wakefield [11:30]: "some of the groups may not have understood what they were talking about. So, if I were to do that again, I think I would definitely make every group go and just write, what they said, or write their formula on the board and start from scratch instead of maybe pointing to somebody else's formula."

Extend

1. How can we find out about students' perceptions of a task (depth, clarity, accuracy)? That is, how do we elicit "good thinking"? How do we know "good thinking" when we see it? What does it look like (e.g., on paper)? What does it sound like (e.g., when working with students during an in-class activity or in office hours)?

2. What kinds of questions will elicit how students are thinking, what they are thinking about, and why they think it?

Transcription of Video Clips

[coming soon]