Inside the Group

Overview

No matter what instructional approaches are used in your class, communication is a very important aspect of teaching and learning. In order for instruction to be effective, students need to understand the teacher, the teacher needs to understand the students, and students need to understand one another. Although mathematical terminology and symbols enable very precise communication, people often use imprecise language to describe mathematical ideas. This imprecision may occur when a student is attempting to learn a new concept, but may also occur when a teacher is attempting to "unpack" a concept that is new to students; this case gives participants opportunities to consider communication issues that arise in the former case.

Learning Goals for Participants

- Consider the role of precision in spoken communication about mathematics.
- Become familiar with one particular approach to introducing antiderivatives.
- Enhance knowledge of student thinking about antiderivatives.

Video Content / Context

The video captures the discussion that occurred as a group of students encountered antiderivatives for the first time. The students were given questions that prompted them to think about how to find a function that has a given derivative.

Activities include listening carefully to a spoken solution of a mathematics task, and inquiring into whether or not members of the group are understanding each other's intended meanings.

Activity Timeline ($\approx$ 45 minutes)

Preview: 10 minutes

View: 15 minutes

Discuss: 10 minutes

Reflect: 10 minutes

Extend: 5-10 minutes per activity

Leading the Case: Inside the Group

Teachers are often concerned about whether students are listening to and understanding what they are saying during class. Clarity of communication is certainly important. Equally important, however, is listening to and understanding what *students* are saying. This can be challenging for several reasons. First, communicating clearly in mathematics is a skill that students are in the process of developing. Second, both students and teachers frequently use pronouns such as “it”, and although the speaker may have a clear idea of what the pronoun refers to, that information may not always be clear to those who are listening. Sometimes this does not cause any miscommunication to occur, but at other times it can generate confusion or hamper peoples’ abilities to understand one another.

The video in this case comes from a calculus class where a group of students is discussing antiderivatives for the first time. During their discussion of how to find particular antiderivatives, the students use the word “it” to refer to a variety of different things. The discussion questions prompt participants to analyze the clarity of the communication and to consider what meaning the speaker and the other students might attribute to “it” during the discussion.

Possible participant discussion points: productive ambiguity, in which sometimes students (intentionally) say things in a non-clear, ambiguous manner so someone else will say it back to them in a clearer, more precise manner.

Possible additional things for participants to think about when watching the video or reading the transcript: pay attention to the correlation (or un-relatedness) between technical words and how the sentence is constructed.

Preview (10 minutes) Participants should read through the worksheet to get familiar with the content of the questions as well as the ordering of the questions. It is important that participants work on and think about each sub-question in question 3. You may need to prompt them to move on and get to that point. $\mapsto$

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Introduction

In this activity, you will watch video of a group of students working on some mathematical tasks. In preparation for viewing the video, you will have the opportunity to look at, think about, and try out some of the tasks.

The discussion questions throughout the activity focus attention on issues of language and precision in communication about mathematical ideas.

This case is taken from a Calculus I class's first experience with antiderivatives. These students have not seen a lecture on antiderivatives, they have not read anything related to antiderivatives, and have done no antiderivative problems. The class regularly engages in activities like the one on this worksheet.

Preview

A worksheet for students in the class is provided below. Read through the worksheet, then discuss the questions on the next page with people in your group and record your group's ideas.

Calculus 1 Worksheet 15

1. Suppose $f'(x) = 3x^2$. What could $f(x)$ be? Check your answer. Are there other possible answers?
2. Suppose $g(x) = x^5$ and $f'(x) = g(x)$. What could $f(x)$ be?
3. When $f'(x) = g(x)$ for all x in an interval I, we say f is the *antiderivative* of g. Find antiderivatives for the following functions.

   (a) $x^5 - 3x^2 + 1$ (b) $2 - \frac{5}{x^2}$ (c) $\frac{1}{2\sqrt[3]{x}}$ (d) $\frac{2}{x}$

Discuss

1. Concepts that participants may bring up: meaning of antiderivatives, finding basic antiderivatives, power functions and trigonometric functions. Possible comments include: notation might play a role. Participants may (or may not) assume that students understand the concept of derivative and function prior to being work on this worksheet.

2. Topics and ideas that may come up: What an antiderivative is and how to find some basic ones, thoughts about "$+c$" when integrating, terminology and symbolism (e.g., $g(x)$, $f'(x)$). Other topics that may come up include the different types of functions that involve the power rule, connections between derivative and antiderivative, and whether students might be learning anything beyond procedure from working the tasks

3. Make sure that participants work through and discuss the antiderivatives of the last two expressions in Problem 3, $\left(\frac{1}{2\sqrt[3]{x}}\right)$ and $\left(\frac{2}{x}\right)$, because these are what the students talk about in the video.

4. Some participants may not have experience taking notes on someone's spoken solution. Writing down what people say, verbatim, may be challenging for some participants. If a group is having difficulty, encourage them to compare their notes and see if they can get as accurate a rendition of the solution as possible.

5. Possible student challenges: fractional exponents; the natural logarithm as an exceptional case for antiderivatives of power functions; x^{-3} instead of $x^{-1/3}$; exponential and trigonometric rules. Some students might think the anti-derivative of x might be 0. Possible participant comments include: Do students know how to check their answers or add "the constant"? There is a lot of language for students to understand or decipher. Possible facilitation issue: people getting caught up discussing the content of the problems and not the issues of student thinking.

Discuss

1. Work through problems 1, 2, and 3 (a, b, c, and d) as a group.

2. Keeping in mind that this is the first time that students have ever seen antiderivative, what are the main mathematical topics in the worksheet?

3. Keeping in mind that this is the first time that students have ever seen antiderivative, what do students have the opportunity to learn by completing these questions in this particular order?

4. Have one person describe a solution to the last part of Problem 3 in a fair amount of detail. While listening to the narration of that person's solution, everyone else in the group writes down exactly what they hear the narrator say.

5. Given that students are seeing antiderivative for the first time, what challenges might they face in doing the last two parts of problem 3? What do you think might be some common student mistakes?

View (15 minutes) Tell participants to detach the last two pages so they can follow the transcript more easily.

Be ready to PAUSE the video: a black screen will appear after line 36 in the transcript. When the video re-starts it backs up a little and restarts at line 30. $\mapsto$

Focus participants' attention on both the words students use in their discussion and the ideas about the mathematics.

Discuss

Glance through the items below, to get a sense of the intended direction of the discussion.

	To what is the speaker referring?	Do others in the group appear to understand the speaker's intent?
(Line 15) Basim: "So it becomes x, x to the zero?"	• antiderivative - you add 1 on exponent • last expression in Problem 3 (x^{-1})	• Yes — maybe? Confused!
(Line 30) Basim: "But x to the -1 is x to the zero."	• what happens when you take the antiderivative of x^{-1}? • the antiderivative of x^{-1} is x^0	• No
(Line 34) Denise: "Yeah, x to the -1 is 1 over x."	• How to define x^{-1}	• Basim doesn't understand • others do understand

View

Watch the video. Pay particular attention to the words the students use in their discussion in addition to ideas about the mathematics they discuss.

A transcription of the focal discussion from the second half of the video can be found at the end of these materials.

Discuss

Refer to the transcript and examine the three excerpts given below. In each case, the speaker is referring to something, but the referent may not be entirely clear. Discuss the questions in the table for each student's statement.

	To what is the speaker referring?	Do others in the group appear to understand the speaker's intent?
(Line 15) Basim: "So it becomes x, x to the zero?"		
(Line 30) Basim: "But x to the -1 is x to the zero."		
(Line 34) Denise: "Yeah, x to the -1 is 1 over x."		

Reflect (15 minutes)
These questions are designed to raise participants' awareness of the possible ambiguities in their own speech, to prompt them to consider productive and not-so-productive types of linguistic ambiguity, and to consider strategies for ensuring good communication with students. ↦

Types of responses from field-testing:

1. People gesture to refer to antiderivative. Some people might say "g", "f-prime" or f' (yes, in referring to antiderivative), use "it" or "f."

2. Someone may know precise definitions but may not know what it means to a student. Students don't speak calculus fluently yet so it's not reasonable to expect them to be clear in their speech. We speak it fluently and so we are making informed decisions about the simplifications we make and we can presume that our colleagues understand us (but can't presume that students will). You can prompt participants to consider how different audiences impact these issues by asking,

   "What are the factors to be considered when deciding how much to simplify your language when talking with students? When talking with peers? When giving mathematical presentations?"

   People may bring up such things as the amount of experience with the content, experience with the particular context, experience with symbols, fluency with various representations, etc. If some students or the instructor are not native speakers of English, that can add another layer to these issues.

3. Possible suggestions from participants: In everyday language we have more tools, for example, pointing, body language, facial expressions to convey meaning. Make things explicit. Talk about differences in how to write things and what they mean, then practice it. We as instructors need to provide opportunities for students to be confused and have the students work through it. Often what I intended to say is different from the students' interpretation. This comes up when writing exams or other materials and it helps to have someone look at your exam, edit it, and give feedback. Students seem to expect you as the instructor to infer what they mean: The instructor is supposed to interpret what the students mean, and students tend to get upset when the instructor does not "relax" explicitness.

Reflect

1. Return to your notes for question 4 from the earlier Discuss section and compare your wording with how the students stated their answers. In particular, in what ways did you refer to antiderivative? (Did you use “it”? Did you refer to the antiderivative using words other than antiderivative?)

2. We use pronouns (such as the “it” used by Basim and Corbin) in everyday language to simplify our speech but doing so may (or may not) introduce confusion. Imagine that you are the instructor in this class and you are listening into the group’s discussion at Line 30 in the transcript. Come up with at least two specific things you could say to the group to accomplish each of the following:

   a) determine *whether* the statements were confusing to other students in the group;

   b) clarify what the speaker meant.

3. Instructors also say and do things that are potentially ambiguous. Generate at least two examples of things that *you* have done while teaching that might not have been clear to all students. What are possible themes or commonalities among the examples generated?

Extend

1. What do these students understand about derivatives and antiderivatives? What evidence can you point to in the transcript to support your claims?

2. Are there other mathematical topics that could be introduced by having students work through a similarly-structured set of problems? What are the potential benefits and drawbacks (for students and for teachers) of introducing topics this way?

3. What can you, as an instructor, do to help students be more precise in their discourse, in addition to speaking and writing precisely yourself? In other words, what can you do while lecturing, while facilitating group work, in writing assignments, in office hours, etc., to help raise students’ awareness about issues of mathematical precision?

4. Issues similar to these come up throughout the undergraduate mathematics curriculum. For each example below, discuss the ambiguity in the statement. What was each student thinking? How might the speaker’s words be interpreted or misinterpreted? How could the speaker have been more precise?

   a) In a Liberal Arts Math class, a student says...

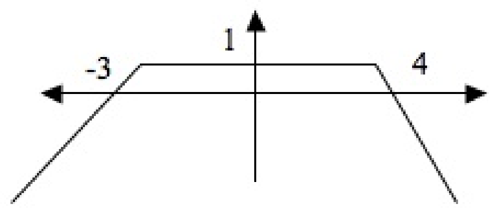
      Rina: We did better on the second exam. The proportion of students who got Ds or Fs dropped by 20 percent.

   b) Pre-Calculus students are working on a problem — Find the roots of the function f pictured below:

Related Resources

Links to other materials:

- **Related Cases: *The Communication Gap*, *Office Hours*, *What Do They Really Get?*** — In each case, students are speaking and using terms they may or may not completely understand.
- **Related Essays: *Group Work* and *Constructivism*** — The essay on group work fills in some background on what research says and the essay on constructivism offers ideas about how we build understanding of concepts.



Pat: Well, it crosses at -3 and 4, so those are the roots.
Lee: And 1, it crosses at 1 too!

c) Calculus I students are practicing the Chain Rule and differentiating

$$y = (3x^2 + x)^4$$

Zach writes his answer and then explains it:

$$y' = 4(3x^2 + x)^3(6x + 1)6$$

Zach: You bring down the four, take one away from it, and then keep multiplying by the inside until you get down to just x.

d) Calculus I students are applying the First Derivative Test to determine if a critical point is a maximum or a minimum.
Dolki: At 3, it's going from positive to negative, so you've got a maximum.

Transcript of Latter Half of Video Clip

1 *Basim*: You gotta go home and rewrite

2 *Corbin*: Nooo.

3 *Basim*: Are you serious?

4 *Corbin*: We're not turning this in.

5 *Basim*: How could you know which one from which?

6 *Corbin*: Cause I wrote it!

7 *Basim*: Oh, okay. Well, *you* know.

8 *Angelica*: Two, x to the negative one.

9 *Basim*: And that's the answer?

10 *Angelica*: No, this is just rewritten.

11 *Basim*: Okay.

12 *Angelica*: And then we can just not use the 2, since it's a constant, and just take the derivative – or the antiderivative – of x to the -1.

13 *Basim*: Which is, zero. You add, on this case you add – antiderivative, you add 1, right?

14 *Angelica*: Add 1, yes. So –

15 ***Basim*: So it becomes x, x to the zero?**

16 *Angelica*: Yeah.

17 *Corbin*: That's interesting.

18 *Angelica*: And then–

19 *Corbin*: You can't use x to the zero, can you?

20 *Angelica*: It's 1.

21 *Basim*: x to the zero is 1, yeah.

22 *Angelica*: Yeah.

23 *Corbin*: But you can use it?

24 *Basim*: Anything to the zero is equal to 1.

25 *Corbin*: Hunh, I know.

26 *Angelica*: I guess.

27 *Corbin*: Except for zero to the zero, right?

28 *Angelica*: Wait. Negative 1 plus 1 is zero. Um–

29 *Corbin*: That's really easy then.

30 ***Basim***: **But x to the -1 is x to the zero.**

31 *Angelica*: Well, x to the -1 is [in unison with Corbin] 1 over x.

32 *Basim*: x to the zer–

33 *Corbin*: What?!

34 ***Denise***: **Yeah, x to the -1 is 1 over x.**

35 *Basim*: But didn't we, didn't we figure it out that we added 1 in here (points to Angelica's paper)

36 *Angelica*: Yeah.
[PAUSE, back up to line 30, and pay careful attention...]

37 *Basim*: And it becomes x to the zero?

38 *Angelica*: Yeah, and that's 1.

39 *Basim*: That's 1, okay.

40 ***Corbin***: Oh, you know what, what if it's, **what if it's um, natural log? Isn't that 1 over x?**

41 *Basim*: Natural log?

42 *Angelica*: Oh!

43 *Denise*: *Natural* log?

44 *Angelica*: So two–

45 *Corbin (to Denise)*: Natural log. There you go.

46 *Angelica*: 1 over x, the natural log

47 *Corbin: Natural log (nods)*:

48 *Angelica*: of x, is 1 over x? The *derivative*! So is it 2 el en of x?

49 *Corbin*: Is the 2 inside? The chain rule? Take it outside later?

50 *Angelica*: It's a constant.

51 *Corbin*: Okay, then, it just carries through. Okay.

52 *Angelica*: Yeah.

53 *Corbin*: Yeah, that works.

54 *Angelica*: Yay!

55 *Corbin*: Whoo hoo!