In: Hauk, S., Speer, N. M., Kung, D., Tsay, J.-J., & Hsu, E. (Eds.) (2013). *Video cases for college mathematics instructor professional development.* Retrieved from http://collegemathvideocases.org/essays/index.php

College Mathematics and Question Strategies

K. Roach, L. Roberson, J-J. Tsay, S. Hauk

"Any questions?" – mathematics instructor most frequently asked question
"   "– most frequent student response

Nearly every instructor at every level asks questions. Some are understood by everyone in the room as rhetorical; others are heard that way by the students, but not meant that way by the instructor. Some elicit a single-word response from a student, providing only a brief respite from an otherwise teacher-dominated lecture. Others spark debates in which the instructor only occasionally chimes in to focus the discussion. Learning how and when to use different types of questions can transform a novice teacher's classroom into a lively, vibrant learning environment.

Kitty's Example

I was a teaching assistant, a TA. As I was preparing for my next Calculus class meeting, my teaching mentor dropped by, curious about what we were covering. I explained that we would be reviewing local maximum and minimum values of functions. My professor laughed and said "Make sure you ask them *why* setting the first derivative to zero will give the possible maximum or minimum values."

I replied, "Of course they know that! We covered it and they didn't have any questions We already had a quiz and they all did well."

He laughed again and said, "Just ask them."

I knew my students understood the concept. After all, we had discussed this in class. They had done homework, and scored well on a quiz. Nobody had any questions about it. I decided to prove to my former professor what I great teacher I was. I got to class and asked the question, "Why do you set the first derivative equal to zero to find the local maximum or minimum values? In other words, why does that work?"

I stood in anticipation, waiting for my students to confirm the greatness of my teaching and their understanding of the concept. One of my stronger students began to explain:

We set the first derivative equal to zero since we know that the function will have a maximum or minimum value when it is equal to zero. So if we set the derivative equal to zero, that will tell us where the function is equal to zero and that gives us our maximum or minimum value.

Wait, what did he say? I could not believe what I was hearing. Not only was he wrong, I couldn't even understand where such an answer would come from. After all, we had talked about the first derivative and how it related to slopes of tangent lines, not to function values. I knew that he must be alone in his thoughts. I looked around the room and saw several other students nodding in agreement. I was shocked.

Before I had a chance to respond another student said, "No, that's not it."

I thought, "Oh good, now we'll hear the correct answer."

He continued, “It’s because when the derivative is zero the graph can go no lower or no higher when we reach our maximum and minimum values, so that’s why the derivative is zero, because the graph can’t be higher or lower than that.”

“Oh, no,” I thought, “it’s getting worse.” and to my astonishment I looked around the room to see students nodding in agreement with this answer as well.

I quickly regained my composure and we took the next few minutes discussing why we actually set the derivative to zero.

I left the room that day completely confused. I had taught that! They had done homework on the topic. They had taken a quiz and done well. I have a friendly welcoming demeanor, and when I asked if there were any questions, students sometimes spoke up – but not about this! Why hadn’t they learned it? Why hadn’t they told me they hadn’t learned it?

That day I began to change as a teacher. It didn’t happen overnight, but I began to change. I realized that by asking questions I could find out what my students were thinking. By changing how I phrased questions, I could help them build their knowledge, slowly understanding Calculus better and better. Often there are connections that I think my students have made but they have not. By asking them questions and giving them time to reflect, some will make those connections. The ones who do not, that's a message to me that we need to address the things I want them to see as connected, perhaps as a class, maybe in office hours.

For many new instructors, knowledge about college mathematics instruction and what kinds of questions to ask come almost exclusively from experience as students in lecture-based courses [11,14]. Researchers have examined the experiences of mathematics graduate students in general [6,7], as well as examining the nature of new college teaching experiences among TAs and mathematicians [2,3,4,16]. These studies shed light on the kinds of questions that are asked and the related classroom and learning consequences. As in other fields, mathematics instructors ask questions to evaluate what students know and to elicit what students think. Here we present two important aspects of questioning, focusing on how the *format* and *context* of a question shapes the kinds of conversation that will (or will not) be generated by it.

After giving some definitions and informal examples, we offer results from an in-depth examination of the use of questions in calculus for five instructors: four graduate student TAs (each was instructor of record for their class) and one experienced professor. We close with a discussion of questioning practices and offer some suggestions for the reader on how to start and expand the use of purposeful questioning.

Initiation, Response, & Follow-up

One model of classroom interaction common in the U.S. is the pattern of initiation, response, and follow-up (IRF) [20]. In college classrooms, this is most often initiated by instructors. In the most common pattern, the teacher initiates with a question (I), a student responds (R), and the teacher follows-up (F) on the student response – often through an evaluation (“yes, you are right”), correction (“not quite, what the next step needs to…”), or restatement (“so what you are saying is…”). The rules for how initiating, responding, and following-up will happen are worked out by the people in the room and or often unspoken or implicit [12]. Examples of the kinds of questions associated with IRF interactions in mathematics are shown in Table 1 (next page).

Evaluate what students know	***Elicit what students think***
Choices – response constrained to agreeing or not with a statement (e.g., Did you get 21?)	***Processes –*** response is an interpretation or opinion (e.g., Why does 21 make sense here?)
Products – response is a fact (e.g., What did you get for #4?)	***Metaprocesses –*** response involves reflection on connecting question, context, and response (e.g., What does the 21 represent? How do you know?)

Table 1. Initiate-Respond-Follow-up (*IRF*) question types and anticipated response types.

Research suggests that U.S. mathematics instructional practice lives largely on the left of Table 1. The unfortunate aspect here is not the fact that basic "evaluate" questions are common but that the other types of questions are uncommon. Questions that elicit what students think can lead to mathematical conversations that foster "mathematical meaning-making" [18, p. 514]. One simple example for turning an evaluate type question into an elicit type of question is replacing "Do you have any questions?" with "What questions do you have?" (accompanied by an expectant and lengthy *wait* for students to generate questions and a *second wait* after prompting for other students to contribute responses, follow-up, or clarifying questions).

Four Levels of Question Strategy

Building on the idea of an IRF cycle, some research has looked at questions in math classrooms by examining classroom discourse - labeled as "math talk" by Hufferd-Ackles and colleagues [8]. Their framework for identifying the type of communication in the room, for both teacher and student, has four categories: questioning, explaining thinking, identifying the source of mathematical ideas, and shaping responsibility for learning. In the "questioning" category the authors describe four forms or "levels" of math talk. Note that in determining level of math-talk, the context of the question is of great importance [15].

To illustrate the importance of context, consider the following question, "Suppose $f'(x)$ is x^5, what could $f(x)$ be?" If asked in a class where students have had repeated practice with the power rule for antiderivatives, this question is a low level recall task. However, if students only have practiced how to take derivatives and have not been introduced to the idea of antiderivative, correctly answering the same question would require a great deal of intellectual work by students. They would need to generate a way to undo the differentiation process. We ask the reader to take as a given that context matters. With this underpinning, we offer the following simplified descriptions of question levels.

Level 0 math talk is characteristic of a traditional classroom in which the instructor's is the primary voice in the room. Only brief answers or responses are required from students. The teacher is the one who initiates interaction and the questions are of the types on the left in Table 1, requiring a yes or no response or a numeric or very brief procedural response. The main goal of the instructor during a Level 0 interaction is getting the right answer into the air in the room.

Questions that might start a Level 0 conversation:

- **What is the answer to problem 5?**
- **If the limit from each side is the same as the value at the point, what do we call the function?**
- **What do we get when we plug in 3?**

In Level 1 math talk, the teacher may pay attention, explicitly, to students' mathematical thinking and focus less on answers alone. However, the teacher is still the center through which communication occurs and most questions are to create an instructor evaluation (choice or product types), with few process questions. The teacher is mainly the one who asks questions (not the students) and follow-up usually dead-ends quickly – once the student has answered a question, the teacher moves on rather than initiating a second interaction based on the student's answer to the first follow up. The main goal is getting the right answer into the room with some attention to how to get the answer.

Questions that might start a Level 1 conversation:

- **How did you do problem 5?**
- **(as a follow-up to student response) No, this function is discontinuous at x=2. Can someone explain why?**
- **I don't see a good way to factor the numerator. Does anyone else have a different way of figuring out what the limit is?**

In Level 2 math talk, a teacher asks some evaluate types of question along with many more eliciting questions that probe the "how" and "why" for the mathematical concepts under consideration. Also at Level 2, the instructor begins to facilitate students talking to each other (e.g., by asking the students to explain their work and their thinking to each other or to report out to the whole class on small group conversations). Common in Level 2 are strings of IRF interactions, mostly teacher-led, where many process and some metaprocess follow-up questions are present. At the beginning of Level 2, the main goal of the questions in IRF-IRF-…-IRF strings may be to get the "right ways of thinking" into the room, while at the upper end of Level 2 the goal includes creating an environment that acknowledges there may be many different ways of thinking that are each productive. Advanced Level 2 math talk shifts to include attention to students building skills at bridging between communicating clearly (to the people in the room) and communicating in mathematically standard ways. That is, in answering a product question like "How did you end up at that answer?" the response of "I FOIL-ed it" is followed or replaced by "I distributed the $u+v$ term."

Questions that might foster a Level 2 conversation:

- **So you said you "FOIL'ed" it – what do you mean by that? How do you know that works?**
- **What about the expression in the integral made you think that integration by parts would be successful?**
- **How is your group's solution similar to or different from theirs?**

In the most complex form of math talk, Level 3, the instructor is co-teacher and co-learner. Students are expected to ask each other about their work and explain their thinking to one another using personal and translated-to-standard-mathematics language without (much) prompting. At Level 3, many initiating questions are "Why?" questions that require justification (in addition to the kind of process or explanation questions seen at Level 2). Also, at Level 3, strings of IRF interaction may occur (as with Level 2) but the purpose of interaction is sense-making rather than getting a particular, favored, way of thinking into the air in the room.

Questions that might foster a Level 3 converstaion:

- **Is this [referring to x^2+6x being an antiderviative for 2x+6] in the family of this [referring to x^2+6x+4]? How do we know?**
- **We've seen two different solutions, but they end up with the same answer. What do you make of that?**

Hufferd-Ackles and colleagues reported on a classroom where communication moved from mostly Level 0 to mostly Level 3 discourse over the course of a year. When the teacher introduced a new topic, she would fold back to a Level 0 or 1 and then rapidly push the discourse to be more complex by eliciting explanations and justifications from students and supporting their use of standard mathematical language with "how" and "why" questions. In Kitty's opening reflection, we sense that previously she had asked lower level questions and she was surprised by her students' understanding when she asked higher-level questions.

Research work around questions in college mathematics instruction conducted by the authors of this essay has indicated that during instruction, we tend to fall into patterns of questioning. For those early in their teaching careers, these patterns may not take *context* into account and may have a math talk approach that forces Level 0 or 1 conversation. For example, Kitty felt she had succeeded by getting the right answer into the air in the room. More experienced and effective college instructors navigate among the Levels based on information they have gathered through evaluating and eliciting types of questions. An interesting finding of research has been that, in the context of group work, an instructor can start math talk at Level 0, rapidly progress through Levels 1 and 2 with questions, and leave a group with a Level 2 or 3 question. That is, to achieve the goal of getting students to own and make sense of the mathematics, it helps to make discussion of ideas the group's responsibility (e.g., with "why" questions to a group that require a conversation). More on this idea is available in the *Group Work* essay (see, in particular, the section on key ingredients for successful group work) and in the *Facilitating Group Work* video case activity.

Finally, though research and development is sparse around questions people ask in teaching Calculus, Miller, Santana-Vega, and Terrell explored "good questions" in college calculus with novice instructors [10]. These questions frequently do not have just one correct answer, or have a popular but incorrect answer. They have the potential to get math talk to higher Levels and can be used to illustrate the larger concepts of the course. Even multiple choice questions, which one might categorize as Level 0, can be used as a launching point to start an engaging learning environment by getting students to defend their different opinions. Used purposefully, any level of question can be a gateway for instructors to guide students to become active participants in the classroom.

Ways to Use Questions More Effectively: Wait Time & Walk-Away Time

Even without changing the wording of a question, what an instructor does after asking a question impacts students' opportunities to engage with the question. Waiting for an answer or walking away with the instruction to students that they decide on an answer amongst themselves are both aspects of question context.

After Kitty's experience, finding out that her students didn't understand the First Derivative Test for maxima and minima as well as she thought, some reflection allowed her to change what she did after asking questions. She later noticed: "Often there are connections that I think my students have made but they have not. By asking them questions and giving them time to reflect, some will make those connections. The ones who do not, that's a message to me that we need to address those connections, perhaps as a class, maybe in office hours."

In our study of question patterns among mathematics instructors, those asking Level 0 questions, including most novice instructors, waited little time (if any) before moving on. In contrast, the most experienced instructor waited a substantial amount of time after even a Level 0 question, and followed it up with eliciting questions that pushed students to discuss possible solutions, different ways to solve, or why to take a particular approach. Many of the studied interactions took place while students were engaged in collaborative group work. In this setting, a Level 0 question rarely prompted more than a few words among the group members. In contrast, when an instructor walked away from a group after asking a "why" or "how" question, the members of the group spent time debating with each other about relationships among mathematical ideas and procedures, working on the problem until they solved it.

In short, higher level questions, and the student engagement they prompted, improved students' opportunities to engage with the mathematics at a deep level. Asking the right questions for the context, and providing space and time for students to grapple with them, helps students unpack what they know, gives them time to communicate with each other about what they know, identify gaps that need to be filled in, and wrestle with the unknown mathematical content and connections.

Next time you are in the classroom notice your questions. The time after that, notice your questions *and* your wait time. Start thinking about the purpose of your questions. To try out the ideas in this essay, prepare a short part of a lesson where you plan so that there is time for you to ask a question, for the students to organize their thoughts, and for you to wait for students to respond (rather than answering the question for them). It is okay to have wrong answers in the room. Not only can they tell you what students are thinking (as in Kitty's case), sometimes those wrong answers are mathematically interesting. Explore the students' suggestions and guide them in the direction they need to go. See where the conversation takes you and your students.

References

[1] Austin, A. (2002). Preparing the next generation of faculty: Graduate school as socialization to the academic career. *The Journal of Higher Education, 73*(1), 24-122.

[2] Belnap, J. K., & Giullian, M. (2008). Teaching discussion among graduate mathematics teaching assistants: Elements contributing to teaching discourse. In M. Zandieh (Ed.), *Proceedings of the 12th annual Conference on Research in Undergraduate Mathematics Education*. Retrieved August 26, 2009 from rume.org/crume2008/Proceedings/Belnap%202%20.pdf

[3] Burton, L. (2004). *Mathematicians as enquirers: Learning about learning mathematics.* Berlin: Springer.

[4] Defranco, T. C., & McGivney-Burelle, J. (2001). The beliefs and instructional practices of mathematics teaching assistants participating in a mathematics pedagogy course. In R. Speiser, C. A. Maher, & C. N. Walter (Eds.), *Proceedings of the 23rd annual meeting of the North American IGPME* (pp. 681–690). Columbus, OH: ERIC (SE 065 164).

[5] Doerr, H. M. & Lerman, S. (2010). Teachers learning from their teaching: The case of communicative practices. In R. Leikin & R. Zazkis (Eds.) *Learning through teaching mathematics.* Springer: Dordrecht.

[6] Golde, C., & Walker, G. (Eds.) (2006). *Envisioning the future of doctoral education: Preparing stewards of the discipline.* Carnegie Essays on the Doctorate. San Francisco: Jossey-Bass.

[7] Herzig, A. (2002). Where have all the students gone? Participation of doctoral students in authentic mathematical activity as a necessary condition for persistence toward the Ph.D. *Educational Studies in Mathematics, 50*, 177-212.
[8] Hufferd-Ackles, K., Fuson, K., & Sherin, M. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*, 81-116.
[9] Kung, D. (2009). Teaching assistants learning how students think. In F. Hitt, D. Holton, P. Thompson (Eds.), *Research in Collegiate Mathematics Education. VII* (pp. 143-169). Providence, RI: American Mathematical Society.
[10] Miller, R. L., Santana-Vega, E., & Terrell, M. S. (2006). Can good questions and peer discussion improve calculus instruction? *PRIMUS 16*(3), 193-203.
[11] National Center for Education Statistics (2002). *Teaching undergraduates in U.S. postsecondary institutions.* (NCES 2002-209). Washington DC: Author.
[12] Nickerson, S. & Bowers, J. (2008). Examining interaction patterns in college-level mathematics classes: A case study. In M. Carlson and C. Rasmussen (Eds.), *Making the connection: Research and practice in undergraduate mathematics education* (pp. 179–190). Washington, DC: Mathematical Association of America.
[13] Seymour, E., Melton, G., Wiese, D. J., & Pedersen-Gallegos, L. (2005). *Partners in innovation: Teaching assistants in college science courses.* Boulder, CO: Rowman & Littlefield.
[14] Sofronas, K. S., & DeFranco, T. C. (2009). An examination of the knowledge base for teaching among mathematics faculty teaching calculus in higher education. In F. Hitt, D. Holton, P. Thompson (Eds.), *Research in Collegiate Mathematics Education VII* (pp. 171-206). Providence, RI: AMS.
[15] Sorto, M. A., Mccabe, T., Warshauer, M., & Warshauer, H. (2009). Understanding the value of a question: An analysis of a lesson. *Journal of Mathematical Sciences and Mathematics Education, 4*(1), 50-60.
[16] Speer, N., & Hald, O. (2008). How do mathematicians learn to teach? Implications from research on teachers and teaching for graduate student professional development. In M. Carlson & C. Rasmussen (Eds.), *Making the connection: Research and practice in undergraduate mathematics education* (pp. 305-218). Washington, DC: MAA.
[16] Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teacher's College.
[17] Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks: Sage.
[18] Truxaw, M. P., &Defranco, T. C. (2008). Mapping mathematics classroom discourse and its implications for models of teaching. *Journal for Research in Mathematics Education*, *39*(5), 489–525.
[19] van Zee, E. & Minstrell, J. (1997). Using questioning to guide student thinking. *The Journal of the Learning Sciences, 6*(2), 227–269.
[20] Wood, T. (1994). Patterns of interaction and the culture of mathematics classrooms. In S. Lerman (Ed.), *The culture of the mathematics classroom* (pp. 149–168). Dordrecht, The Netherlands: Kluwer.