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Group Work in College Mathematics Classes

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Imagine yourself walking down the hallway of a large university's mathematics department. Walking by the largest lecture hall, you hear Professor Abel giving instructions to his Linear Algebra class. "OK, you see that the class voted pretty much evenly for all four answers. I think it's fairly clear that only one of them can be correct. Take a few minutes to talk to the students around you and see if you can come to an agreement about which is correct."

You walk a little further down the hall and peak in on a Calculus discussion section. All 25 students are crowded in groups along the whiteboards. The room is buzzing with activity with each group of three or four students arguing their way through a worksheet full of problems. The teaching assistant – you think you know which one is the TA – wanders around the room, listening in on a group's chatter and occasionally interjecting a question or comment.

Finally, you reach the end of the hallway where you know that Professor Cantor is teaching an upper-level Logic course. From the hallway, you hear him finish a proof and start in on the last item on the day's agenda. "That finishes the main portion of the semester's material, and brings us to the last two weeks when you'll be giving the group presentations that you've been working on all semester. I've met with every group at least twice now, so I know you're making good progress. I just wanted to give you the last ten minutes of class today to finalize your plans for the next few days and ask me any last questions."

In your short stroll down the hallway, you have observed three very different implementations of what one researcher calls "one of the greatest success stories in the history of educational research" (Slavin, 1996) namely, cooperative learning. Group work, or cooperative learning, has been studied in classrooms at all levels from elementary school through graduate school. The research literature on such teaching strategies is vast and even if restricted to collegiate mathematics and science classrooms, would cover hundreds of studies (Springer, Stanne, & Donovan, 1999). This literature shows that incorporating group work in college mathematics courses has the potential to improve students' academic achievement, attitudes toward mathematics, and persistence in mathematics-related fields. Research also clearly shows that simply asking students to work together is unlikely to produce such positive outcomes. Here we give a brief, and necessarily incomplete, overview of this body of literature, concentrating on those aspects most applicable to the college mathematics setting.

We begin by discussing what is meant by the term group work (or cooperative learning, or collaborative learning, all of which are used interchangeably here), examining the wide variety instructional strategies that fall into this category. We then give an overview of the research comparing group work with more traditional instruction. These studies answer the question, "Why use group work?" by documenting gains in a variety of measures of achievement and attitude. Next we turn to an analysis of what particular aspects of group work have been found to be necessary to achieve student gains. We then give an overview of the various theories which researchers have developed to explain why cooperative learning strategies are so effective. Finally, this essay is not intended as a guide to implementing group work in the classroom. Several such guides specifically for college mathematics have been written, including one by Davidson (1990) and one published by the Mathematical Association of America (Rogers, Reynolds, Davidson, & Thomas, 2001).

What is Group Work?

As your walk down the hallway indicated, group work can be implemented in a wide variety of ways. Over 100 different forms of cooperative learning have been identified at levels from kindergarten through college (Sharan, 1994). This abundance has led to a potentially confusing situation, with varying definitions of group work and the drawing of distinctions among subsets of ideas like *group work*, *collaborative learning*, and *cooperative learning*. For the moment, in this short report, we ignore such fine-grain distinctions and use any of these terms to refer to students working together on a common task with the goal of learning academic material.

At the college level, especially in mathematics and science classes, there has been a recent renewed interest in group work. During the late 1980s, the Calculus Reform movement, in part funded by the National Science Foundation (NSF) included a number of pedagogical reforms, with collaborative learning prominent among them (Tucker & Leitzel, 1995; Tucker, 1990). For some innovators this was a conscious choice. Others were pushed to have students work in groups when other features of instruction made it necessary such as not enough computers being available or when instructors' workloads became unwieldy (Brown, 1996; Hagelgans et al., 1995; Ross, 1996). Even within the context of college Calculus, the most frequently studied college mathematics course, the form that group work takes varies significantly from one school to another and from one classroom to another. Here we give some examples, concentrating on how group work is used in different settings. To allow for a comparison of these models, we focus exclusively on studies of Calculus courses. A similar breadth of uses of group work could be selected at many levels, from Pre-calculus to Abstract Algebra, and at many institutions from community colleges to elite universities.

Duke University was among the first institutions to receive NSF funding for calculus reform. Begun in 1988, Project CALC (Calculus As a Laboratory Course) featured "real-world problems, hands-on activities, discovery learning, writing and revision of writing, teamwork, and intelligent use of available tools" (Bookman & Blake, 1996). In particular, a weekly lab was added in which students worked in pairs, using computers to investigate complicated calculus concepts. In addition, the three regular weekly class meetings were restructured from being mostly lecture to including a significant amount of group work, where students worked on substantial problems and summarized their results in biweekly written reports (Smith & Moore, 1990).

When the University of Michigan restructured their calculus courses, Morton Brown included a more limited use of group work (Brown, 1996). Using a reform textbook (Hughes-Hallet & Gleason, 1992) and graphing calculators, instructors at Michigan incorporated both group homework and cooperative learning in the classroom. Among the listed program goals were students becoming better lifelong learners by working cooperatively, and improving instruction with a student-centered approach and instructor-facilitated cooperative learning (Burkam, 1994).

At Cornell, Maria Terrell has pioneered the use of multiple choice questions in both small and large sections of Calculus (Miller, Santana-Vega, & Terrell, 2006). Called *Good Questions*, this program is sometimes used in conjunction with "clickers," small electronic devices that allow students to register their answer and have the class distribution displayed in near-real-time (e.g., projected from computer to a screen at the front of a lecture hall). In some lectures, students are asked to discuss their answers before and/or after clicking in, leading to either a full class discussion or a re-vote. In addition to the original proprietary clicker devices, smart phones and text messaging systems are now used to gather student input in real time.

At Ursinus College, Nancy Hagelgans creates groups that work together for the entire semester in a variety of settings (Hagelgans, 1999). While many of the assessments used throughout the semester are traditional, individual exams, the first mid-term exam is a group test. Even on the individual exams, if all members of a group do well on an individual exam, they may earn extra credit.

Following the lead of Uri Treisman, colleges and universities around the country have instituted Emerging Scholars Programs, designed to help underrepresented groups excel in Calculus (Hsu, Murphy, & Treisman, 2008). In a typical Emerging Scholars Program, students might attend an additional four hours of class facilitated by an advanced undergraduate or graduate student teaching assistant each week. During these sessions they work with their classmates on a set of challenging problems. These workshops are typically graded only on effort and participation, with all formal assessment occurring in the students' regular sections – which they also attend.

These are only a few examples of the myriad ways college Calculus courses have used group work. Why have so many instructors chosen to incorporate collaborative learning strategies into their classroom? What evidence do we have that such strategies work? The next section tries to answer these questions.

What are the Effects of Using Group Work?

The remarkable success of group work, and why Slavin calls it one of educational research's "greatest success stories" is that the positive benefits of having students work together are remarkably resilient to changes in the implementation. While different instructors use different types of tasks and assessments, the general trend still holds: students in collaborative settings do the same or better than students in more traditional settings (Slavin, 1996; Springer et al., 1999).

Here we review the research basis for these remarkable claims, looking both at claims of improved achievement and improved attitudes. In the subsequent section we address the much harder question of how the various instructor implementations of group work affect outcomes.

After studying several meta-analyses of group work (which combine the results of a large number of separate studies), Bossert (1998) claimed that

> In general, the research has successfully demonstrated that student achievement is at least as high, and often higher, in cooperative learning activities as in traditional classrooms. At the same time, cooperative learning methods seem to promote positive interpersonal relations, motivation to learn, and self-esteem among students. Recent meta-analyses suggest that the benefits of cooperative learning activities hold for students at all age levels, for all subject areas, and for a wide range of tasks, such as those involving rote-decoding, retention, and memory skills, as well as problem-solving ability.

The meta-analysis most applicable to the college mathematics setting limited its scope to undergraduate mathematics and science courses. Published in 1999, Springer, Stanne, and Donovan combed the research literature for examples of studies done since 1980 comparing collaborative learning methods with more traditional ones (Springer et al., 1999). After examining the resulting 383 studies and restricting their scope to classroom-based studies with results that included a measure of effect size, 39 separate studies were suitable for inclusion in their meta-analysis. These 39 studies represented a wide range of the different implementations of group work. To compare different studies, the authors used the standard mean difference (d-index) effect size (Cohen, 1994). For such quasi-experimental studies, this would be the experimental group's average score minus the control group's average score, divided by the average of the two groups' standard deviations.

Spring and colleagues' (1999) meta-analysis concluded that "the main effect of small-group learning on achievement, persistence, and attitudes among undergraduates was significant and positive." (p 29). The average effect sizes, over all the data in the 39 studies, included significant increases in acheivement (d=0.51), persistence (d=0.46), and attitudes (d=0.55). To put this in practical terms, an effect size of 0.51 "would move a student from the 50th percentile to the 70th percentile on a standardized (norm-referenced) test." (p. 38). In terms of other outcomes, group work was correlated with improved attitudes toward the course material ($d = .56$), and with self-esteem ($d = .61$). Persistence in science and mathematics majors was also positively correlated with cooperative learning strategies ($d = .46$).

Studies done at the K-12 level indicate a whole host of other positive effects of having students work together. In addition to the improved achievement, attitudes, and persistence found in college classes, studies indicate that group work in primary and secondary school enhances student self-esteem, promotes racial and ethnic integration, improves attitudes toward school in general, improves students' time on task, and improves student behavior (see Slavin, 1991, for an overview of this literature.)

What are the Key Ingredients in Promoting Successful Group Work?

While in general group work is correlated with positive effects, individual instructors' choices as to how a particular version of group work is implemented might have drastic impacts on its effect. As the introduction to this essay indicated, group work can take many different forms at the college level. Which aspects are necessary to promote the gains mentioned above? Which aspects are sufficient?

Research indicates that subtle changes in implementation of a teaching method can produce large changes in outcomes. Such cases have been well-documented. For instance, studies show that in Physics courses using clicker questions (similar to the *Good Questions* project described above), students who discuss their answers before the correct answer is revealed learn significantly more than those who do not (Crouch & Mazur, 2001). Early studies of the *Good Questions* project indicate that a similar pattern happens in mathematics courses: students who answered the questions and discussed their answers with peers outperformed those who just answered the questions (Miller et al., 2006).

While not enough research has been conducted at the college level to conclude definitively what promotes successful group work, we can extrapolate from work done at the K-12 level. On this point, the most experienced researchers have drawn conclusions that broadly agree on some points and differ on others. Here we present a brief overview of this research.

Robert Slavin has been studying the effects of group work for three decades. In his analysis of the research literature, he concludes there is a fairly strong consensus among researchers that two features are essential in promoting positive outcomes in the U.S.:

(1) clear group goals,
(2) individual accountability to the group as a whole.

By group goals, he means that groups must work together to earn some form of group success, which might range from a good grade on a project to public recognition by a teacher. The second necessary feature, individual accountability to the group as a whole, means that the group's success must depend on each individual learning the material. According to Slavin, "studies of methods in which students work together to prepare a single worksheet or project without differentiated tasks hardly ever find benefits." (1991, p. 100).

Additionally, brothers Roger and David Johnson have been studying group work and collaborative learning for over four decades. Their work indicates that five elements are critically important in getting students to productively cooperate (Johnson & Johnson, 1987; Johnson et al., 1998). These five vital parts of group work are the following (paraphrased from Johnson et. al., 1998):

Positive interdependence. Each student should perceive that she is linked with others in such a way that the student cannot succeed unless others do. Possible mechanisms include joint rewards (for all group members doing sufficiently well), divided resources (e.g. only one worksheet for the group), and complementary roles (timekeeper, reporter, elaborator, skeptic, etc.)

Individual accountability. One goal of group work should be students learning together so that they can subsequently perform better as individuals (in giving an explanation, in presenting to the class, or on individual assessments.)

Positive interactions. Working in groups small enough to forge meaningful, face-to-face interactions (two to four members), students should promote each other's success through helping, supporting, encouraging, and praising one another's efforts.

Social skills. The interpersonal skills needed to work effectively with others (leadership, trust-building, conflict management, etc.) are learned behaviors and should be taught alongside academic skills.

Group processing. Groups that take time to analyze the group's processes and identify possible improvements will ultimately become more efficient, more effective at working together.

In contrast, veteran researcher Elizabeth Cohen argues that if an instructor chooses the group task well, then effective group work does not necessarily depend on having positive interdependence in the reward structures (Cohen, 1994). In her "inductive and conceptual review of research" (p. 2) her analysis of the literature stresses the importance of "true group tasks." Facing a task that "requires resources (information, knowledge, heuristic problem-solving strategies, materials, and skills) that no single individual possesses so that no single individual is likely to solve the problem or accomplish the task" (p. 8) promotes the types of positive, task-oriented interactions that help students overcome conceptual barriers. When students work on true group tasks, Cohen goes on to conclude, the sheer volume of interactions becomes a fairly good predictor of learning.

As the above reviews indicate, different approaches have led to slightly different conclusions about what features of group work are critical to its success. While these ideas do provide insight into what college mathematics group work assignments might be most productive, there is also evidence that not many college instructors implement group work in ways that abide by these suggestions (Colbeck, Campbell, & Bjorklund, 2000).

Why Does Group Work Work?

A number of different learning theories have been used to explain why collaborative effort enhances student learning. Several of these are discussed in greater length elsewhere in this volume. The history of collaborative learning weaves together these strands in complicated ways. Here we give a (necessarily simplified) view of how each theory answers the question, "Why does group work work?" .

Constructivist Theory. Based on the pioneering work of Piaget and Vygotsky (Bidell, 1988; Smith, Dockrell, & Tomlinson, 1997) constructivist theories are grounded in the idea that the process of learning involves humans constructing and refining mental structures to organize and reorganize existing knowledge and ultimately incorporate new information. From this perspective, group work activities such as discussions, debates, and explanations force individuals to confront confusion and construct new, improved mental images when engaging with new concepts. Through a cognitive conflict between an existing way of thinking and new information, mental re-structuring occurs and the new structure is retained, to be used in later situations (and as the basis for later restructuring).

Social Interdependence Theory. Social structures impact, and some say even determine, how individuals act and react. If we understand how individuals react to various social structures, we can predict the interactions and outcomes, according to this theory, based in the work of Koffka (1935), Lewin (1935), and Deutsch (Deutsch, 1949). In terms of group work, positive interdependence (cooperation) results in positive, promotive interactions between individuals. In contrast, negative interdependence (competition) promotes negative, oppositional interactions as individuals work to undermine each other. For social interdependence theorists, this explains why in many studies that compare cooperative, and competitive learning, the cooperative group fares best.

Motivational Theory. Several different theories focus attention on the motivations of the learner. These range from Skinner's behaviorist perspective (Skinner, 1976) which would focus on the extrinsic motivators (e.g., rewards and grades) to those who focus on intrinsic motivators such as the desire to succeed (Deci & Ryan, 1985). From this perspective, the reward structures, in terms of grades, acknowledgement, and social rewards, are the keys to understanding the effects of working in groups.

Conclusion

Educational researchers still have much to understand about group work, especially at the college level. What is clear from the current literature, however, is that having students work in groups – for a few minutes of discussion or for a semester project – has the potential to improve many aspects of student learning. By looking more closely at the ways college mathematics teachers implement group work, paying attention to the specifics of the assigned tasks and the accountability structures, the advantages of group work are more likely to be realized.

Many of the video cases in this volume touch on college mathematics instruction that incorporates group work. In particular, there are three that directly address the topic: *Facilitating Group Work*, *Inside the Group*, and *Choosing and Ordering Student Work*.

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