

## Constructivism

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Constructivism is a philosophy about how we gain knowledge. It is based on the premise that people build mental models to make sense of their world. Strictly speaking, constructivism is not a teaching theory, but a theory regarding how the mind works. In this way, it can be regarded as learning theory, for one product of a mind's working is knowledge gain (learning). Pedagogies or teaching models that are called "constructivist" are based on an interpretation of constructivism in the context of teaching application – including this essay. Although it is not a theory of teaching, constructivism does suggest a different approach to instruction from the lecturing used in most undergraduate service courses like pre-calculus/college algebra, calculus, and pre-service K-12 teacher courses.

John Dewey is often cited as the philosophical founder of constructivism. At the turn of the 20<sup>th</sup> century, Dewey directed interest from "theory of knowledge" to "theory of inquiry" and asserted that to improve teaching and learning it was important to treat thought as if it existed separately from the then-current notions of "reality"<sup>1</sup> (Field, 2006). In other words, constructivist views of knowledge focus not on a presumed "real" world but on our *perceptions* of the world. In particular, two ways of looking at how we construct our understanding of the perceived world have emerged. *Individual* or *radical constructivism*, guided by the thinking of Jean Piaget (1954), centers on the way individuals create mental structures. *Social constructivism*, based on the theories of Lev Vygostky, takes a slightly different approach by focusing on the significance of humans as social, interactive, thinkers. The following discussion of constructivist theories of learning and possible teaching applications is based on these two perspectives.

### Individual Constructivism

With a background in biology, Piaget was interested in how organisms, including human beings, *adapt* to their environment. Piaget portrayed the adaptation as driven by a biological drive to obtain balance (i.e., "equilibration") between our external stimuli and our mental structures or *schemes*. Piaget described two processes used by an individual in the attempt to adapt and achieve equilibration: *assimilation* and *accommodation* (Furth, 1981; Driscoll, 1994). *Assimilation* is the process of mentally transforming the perception of external stimuli so that it can be fitted in preexisting schemes. For example, putting three more apples into a basket that already contains five apples can be assimilated to the scheme "5+3." Rather than fitting a newly perceived and disequilibrating idea into

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<sup>1</sup> "Theory of knowledge" is a branch of philosophy called epistemology. Dewey's redirection toward a "theory of inquiry" was trying to avoid a then-common debate of a distinction between thought and its corresponding world of fact, a debate especially critical between rationalists and empiricists.

existing schemes, *accommodation* is the process of revising or creating new schemes in order to satisfy external stimuli. A very basic example would be considering a basket with five apples and being asked: How many more apples are needed for the basket to have a total of eight? The scheme for  $5+3$  might have to be modified to include the possibility of counting up from 5 and getting a result. The new scheme might be represented by “ $5+[\ ]=8$ .”

Both assimilation and accommodation are valuable as models for learning. Humans are pattern seekers. That is the heart of assimilation – recognizing and identifying with a known pattern. In addition to identifying and storing ideas, people also create their own interpretations of ideas based on their perceptions – this is the heart of accommodation.

To illustrate these ideas, we note that mathematical rules as stated and organized on paper must somehow make their way into human minds. The numerous inconsistencies in mathematical representation are just one source of challenge in this endeavor. A student who is trying to make sense of a property of integers, “ $-(-a) = a$ ,” for example, may transform this property and then assimilate such a transformation into any one of several familiar schemes: one useful assimilation would be to connect double negation to pressing a button once and then pressing the button once again to *cancel* the former action. Later, on seeing the property, “ $(-a)(-b)=ab$ ,” the same student might not see the button-pushing scheme as appropriate, because the negative signs in the statement are no longer consecutive. The learner then faces cognitive conflict between an effort to assimilate the new idea into an old scheme and the fact that the old scheme does not seem to fit. To resolve the cognitive disequilibrium, the learner may create a new scheme to accommodate the fact that the two negative symbols are not immediately adjacent to each other. Research on student learning suggests that frequently the simplified symbol-based schema of “two negatives is a positive” is the result.

While procedurally useful, this type of accommodation often proves problematic for learners as they encounter more complex demands in thinking about signs and operations. In particular, students run into difficulty (e.g., disequilibrium) attempting to assimilate new ideas to the “two negatives is a positive” scheme when working with exponential expressions like  $-2^{-1}$ . Assimilation would treat  $-2^{-1}$  as if the statement were  $(-2)(-1)$ . That is, the external stimulus of  $-2^{-1}$  is changed so that an existing scheme can be used. Nonetheless, the highly simplified “two negatives is a positive” scheme reduces the intricacy of mathematical tasks, lessens learners’ stress about loss of equilibrium, and is common among most novice learners. By creating a new mental structure for the “two negatives is a positive” scheme, the student has learned. But, have they learned what was intended?

Similarly, as instructors we might assimilate in counter-productive ways. Consider the instance where a pre-calculus instructor is talking about “how to solve”  $3+bi = (7+6i) - (4+12i)$  and a student who is still struggling with determining when the letter  $i$  is *treated as* a variable, asks “How would we solve for  $i$ ?” The teacher answers by solving the equation for  $b$ . The student repeats the question, and the teacher says “but I just showed you!” The teacher assimilated the student’s question into one that made sense to him in the context, “how do we solve for  $b$ ?” (rather than “for  $i$ ”), and answered the assimilated question.

A preferable scheme in mathematics to the “two negatives is a positive” scheme, could be: the product of the additive inverse of  $a$  and the inverse of  $b$ , “ $(-a)(-b)$ ” is equal

to the product of  $a$  and  $b$ . That is, students could accommodate by creating a scheme that is exactly the “spelling out” of the rule. For students to be able to construct (or reorganize/re-represent) such a scheme, they first would have to perceive a need, see that the existing scheme is not sufficient for sense-making of external stimuli, such as a failed attempt at assimilation. Then, the reorganization/re-representation of the existing schemes would be possible. In this given case, expand whole numbers by incorporating additive inverses and investigating all the possible interactions among these elements.

A constructive accommodation, when it happens, involves a change in mental structure. Though learning is often the result, such changes are not always appropriate adaptations. That is, research asserts that learning proceeds through construction, rather than straightforward “absorption” of knowledge (Romberg & Carpenter, 1986). An inappropriate assimilation or accommodation can lead to what is called by some a “mis”conception. This is a conception held by an individual that is judged to be false by those who set the standards for a concept. Research has demonstrated the stubbornness of such unproductive (or counter-productive) adaptations. These adaptations may be sufficiently useful in the face of disquilibrium that they become deeply ingrained in the mental map of an individual and end up unlikely to be abandoned.

The notion of a *misconception* requires consideration of students’ knowledge and experience in designing learning tasks, for students may not learn the way instructors intend. Some educators have emphasized careful design of activities to elicit cognitive conflict with common misconceptions. That is, in the language of Piagetian constructivism, by setting up a mathematical problem situation to evoke disequilibrium in students, instructors can encourage the construction of new, complex, and appropriate mental schemes. (Novak, 1987).

The processes of assimilation and accommodation are used as a person adapts to the environment and constructs progressively more complex schemes. As schemes become increasingly complex, they become structures and are organized in both networked and hierarchical ways. Hierarchically in mathematics, these can be expressed in terms of the building blocks of definitions, axioms, and theorems – all held together by the mortar of logical consistency. Traditional methods of college mathematics curriculum and instruction are based on this hierarchy: students master several sub-skills, often not clearly connected to each other, to prepare for the eventual orchestration of a complex scheme. This perspective heavily emphasizes what has been constructed, but ignores how knowledge is constructed and schemes are connected.

From the individual or “radical” constructivists’ viewpoint, thinking processes<sup>2</sup> are as important as the content about which the thinking occurs, how else are learners able to connect new skills to existing schemes (Brooks & Brooks, 2001)? Both individual and social constructivist theorists agree, learning is facilitated when learners are offered the chance to engage in mathematical activities that are truly problematic for them and when mathematics is offered to be learned as a whole, rather than as practice of a disconnected set of algorithms and sub-skills (Boaler & Greeno, 2000).

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<sup>2</sup> Examples of thinking processes include framing and solving problems, looking for patterns, making conjectures, examining constraints, making connections, reasoning and justifying, making inferences from data, inventing, explaining, challenging, etc.,

### Social Constructivism

Given the current disjointedness of most school and college mathematics curricula, mathematical activities with high cognitive demand are perceived by most students as ill-structured and ambiguous. It is not apparent to learners what they should do and how they should do it. Consequently, mathematical activities are likely to decline into less-demanding activities (Stein, Grover, & Henningsen, 1996). Immediate support in the form of temporary framework can be given to students in order to help them achieve success in a task they could not otherwise accomplish without assistance. When such support is then gradually removed, the student can become a self-regulated, independent learner and problem-solver. This carefully designed instruction, called *scaffolding*, is inherent in Vygotsky's idea of the Zone of Proximal Development (ZPD).

Having a strong belief in culture as the prime determinant of individual development, Vygotsky's perspective might be seen to extend the Piagetian view of cognitive development from the "actual developmental level" to the "potential developmental level." The Zone of Proximal Development is what people think about and talk about that is just on the edge of what they, as individuals, already know – the region between the actual developmental level of people as independent problem solvers and the level of potential development of a group of individuals guided by a more knowledgeable other. If three students are tackling a challenging problem, each will have a personal ZPD and there will be a SPD for the group. Social interaction with a more knowledgeable other, such as a teacher or student-peer, particularly through verbal and pictorial discourse, is how an interpersonal process is gradually internalized into an *intrapersonal* one (Vygotsky, 1978). Schoenfeld (1992) refers to this type of social interaction, as "enculturation." For this enculturation, the individual is immersed in an established mathematical culture and gradually picks up and internalizes into a kind of personal dialogue with the self the *accepted norms and values* of mathematics. For example, the learner of mathematics may begin to see the world like mathematicians do. Or, the "socio-mathematical norms" may be for a different culture (e.g., mathematical physics or engineering or teaching). For more on *socio-mathematical norms*, see Yackel, Rasmussen and King's (2000) investigation of norms in a college differential equations course.

As mentioned, the property, " $-(-a)=a$ ," formally interpreted by a learner as "two negations canceling each other" is not an accepted norm in mathematics society. A learner may be able to give a formal explanation after mimicking an instructor's demonstration: by definition of the additive inverse,  $a+(-a)=0=-(-a)+(-a)$ , by the uniqueness of the additive inverses, it must be that  $-(-a)=a$ . Nonetheless, most students find it prohibitively challenging to expand this to demonstrate the property  $(-a)(-b)=ab$ . Very likely, students, initially, do not have a "uniqueness of the additive inverses of integers" scheme into which to assimilate or revise to accommodate  $(-a)(-b)=ab$ .

Vygotsky's social constructivism suggests that a more knowledgeable other can evaluate what learners do at their current development level, build a scaffolding to assist them to master advanced skills at their highest potential level, and gradually remove the supportive framework. During this process, students need to gradually take on more responsibility (Hogan & Pressley, 1997). In the example of providing acceptable explanation for students to internalize the property " $-(-a)=a$ ", tasks such as "find all the forms of additive inverse for  $-a$  and justify your answer" can elicit guesswork and

intuitive thinking in learners. Through observation and discussion of what students derive, a decision might be made by putting  $a+(-a)=0$  and  $-(-a)+(-a)=0$  together to draw a conclusion,  $-(-a)=a$ . In closure, questions such as “what makes these two pieces of information useful to the conclusion  $-(-a)=a$ ?” can challenge learners to reflect on the processes of this task. For a new task, such as justifying the property  $(-a)(-b)=ab$ , asking students how to identify sub-problems may support more responsibility for their learning.

### Conclusion

Several teaching models based on constructivism have been exemplified as effective and enjoyable ways of learning mathematics. These include activity-based, student-centered, and reciprocal learning, (see, e.g., Bourdon and Carducci, 2002 or Carlson and Rasmussen, 2008, and references therein). These models are different from lecture-based approaches for covering the scope of content. Rather than requiring that all ideas be uttered by the instructor in order for students to learn, constructivist-informed approaches recognize the diversity of existing mental structures held by students when they walk into the classroom. Under constructivism, students are not “blank slates” to be written on by teacher presentations. Instead, students are construction workers in their own mental processes and teaching is providing opportunities for each student to re-structure, build, and re-organize their thinking in ways consistent with the intended mathematical content (Cobb, 1994).

In the video cases included in this volume, the scaffolding of social constructivism is evidenced in the cases: *Leading Whole Class Discussions*, *Facilitating Group Work*, and *Inside the Group* while food for thought about the assimilation and accommodation of individual constructivism can be seen for students in the cases *What Do They Really Get?*, *Choosing and Ordering Student Work*, *Office Hour*, and for instructors in *Grades and Grading*.

### Extensions

Jot down your responses and then discuss with some teaching colleagues.

1. What teaching have you experienced *as a student* that seems to align with constructivism? Give specific examples from that experience (at least two) and describe how these examples are evidence of a constructivist approach to teaching and learning.
2. What opportunities have you experienced as a learner to work with others to build understanding in your Zone of Proximal Development? Carefully describe at least one instance that seemed consistent with the ideas of social constructivism; in what way(s) does the experience seem to align with social constructivism? Was it an effective learning experience? How do you know?
3. Read either (a) an article from Carlson & Rasmussen (2008) OR (b) Yackel, Rasmussen, and King (2000) and identify at least one way your teaching is similar to the instruction described in the article and at least one way it is different. In what ways are these two aspects of instruction consistent with constructivist theories? In what ways may they be inconsistent with a constructivism-informed approach to instruction? Relate at least two specific pieces of evidence from the reading to support your answer.

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